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Reg. No.

V Semester B.Sc. Degree Examination, April - 2022

MATHEMATICS

(CBCS Scheme Semester 2022)

Paper : V

Time : 3 Hours **Maximum Marks : 70**

Instructions to Candidates:

Answer all questions.

PART - A

Answer any **five** questions. **(5×2=10)**

1. a) In a Ring $(R,+,\cdot)$ prove that $(-a).(-b)=a.b\ \forall a,b\in R$.

   b) Define subring of a ring and given an example.

   c) Prove that every field is a principal ideal ring.

   d) Find the maximum directional derivative of $\phi = x^3y^2z$ at the point (1,-2,3).

   e) If $\vec{F} = yz\hat{i} + zx\hat{j} + xy\hat{k}$ then show that $\vec{F}$ is irrotational vector.

   f) Evaluate : $\Delta^4(1-ax)(1-bx)(1-cx)(1-dx)$.

   g) Write the Newton's divided difference interpolation formula.

   h) State the Trapezoidal rule for the integral $\int_a^b f(x)dx$.

PART - B

Answer **two** full questions. **(2×10=20)**

2. a) Prove that the set $R=\{0,1,2,3,4,5\}$ is a commutative ring with respect to $\oplus_6$ and $\otimes_6$ as the two composition.

   b) Prove that a ring R is without zero divisors if and only if the cancellation laws hold in R.

OR

3. a) Prove that the ring $(Z_n, +_n, X_n)$ is an integral domain if and only if n is a prime number.

   b) Show that the set of all real numbers of the form $a+b\sqrt{2}$ where a and b are integers is a ring with respect to addition and multiplication.

[P.T.O.

4. a) Find all the principal ideals of the ring $R = \{0,1,2,3,4,5,6,7\}$ with respect to $\oplus_8$ and $\otimes_8$.

b) If $f : R \to R'$ be a homomorphism with kernal k, then prove that f is one - one if and only if $k = \{0\}$.

(OR)

5. a) Let $R = R' = C$ be the field of complex numbers, Let $f : R \to R'$ be defined by $f(Z) = \overline{Z}$ where $\overline{Z}$ is the complex conjugate of Z, show that f is an isomorphism.

b) State and prove fundamental theorem of homomorphism of rings.

PART - C

Answer **two** full questions. **(2×10=20)**

6. a) Find the directional derivative of $\phi(x, y, z) = x^2 - 2y^2 + 4z^2$ at the point (1,1,-1) in the direction of $2\hat{i} - \hat{j} + \hat{k}$.

b) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$ then prove that $\nabla r^n = nr^{n-1}\hat{r}$.

(OR)

7. a) Find the angle between the surfaces $4x^2y + z^3 = 4$ and $5x^2y - 2yz = 9x$ at the point (1,-1,2).

b) If the vector $\vec{F} = (3x + 3y + 4z)\hat{i} + (x - ay + 3z)\hat{j} + (3x + 2y - z)\hat{k}$ is solenoidal find 'a'.

8. a) Prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$ where $r^2 = x^2 + y^2 + z^2$.

b) Show that the vector $\vec{F} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational. Find ϕ such that $\vec{F} = \nabla\phi$.

(OR)

9. a) Find Curl (Curl $\vec{F}$) where $\vec{F} = x^2y\hat{i} - 2xz\hat{j} + 2yz\hat{k}$.

b) If ϕ is a scalar point function and $\vec{F}$ is a vector point function then prove that $div(\phi\vec{F}) = \phi div\vec{F} + grad\phi.\vec{F}$.

PART - D

(2×10=20)

Answer **two** full questions.

10. a) Use the method of separation of symbols to prove that

$$U_0 + U_1x + U_2x^2 + \ldots\ldots + \infty = \frac{U_0}{1-x} + \frac{xU_0}{(1-x)^2} + \frac{x^2\,\Delta^2 U_0}{(1-x)^3} + \ldots\ldots\infty .$$

b) Obtain a function whose first difference is $x^3 + 3x^2 + 5x + 12$.

(OR)

11. a) Find a cubic polynomial which takes the following data.

x	0	1	2	3
f(x)	1	2	1	10

b) Find f(9.7) from the following data

x	8	8.5	9	9.5	10
f(x)	50	57	64	71	75

12. a) Using Lagranges interpolation formula find f(2) from the following data

x	0	1	3	4
f(x)	5	6	50	105

b) Using Simpson's $\frac{3}{8}^{th}$ rule Evaluate $\int_0^6 \frac{1}{1+x^2}dx$.

(OR)

13. a) Evaluate $\int_0^{0.6} e^{-x^2} dx$ by taking 6 subintervals By using Simpson's $\frac{1}{3}^{rd}$ Rule.

b) Prepare divide difference table for the following data.

x	1	3	4	6	10
f(x)	0	18	58	190	920