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Reg. No. | | | | | | | | |

V Semester B.Sc. Degree Examination, April - 2022

MATHEMATICS

(CBCS Semester Scheme)

Paper : VI

Time : 3 Hours **Maximum Marks : 70**

Instructions to Candidates:

Answer all questions.

PART - A

Answer any **five** questions. **(5×2=10)**

1. a) Write Euler's equation, when f is independent of y.

   b) Show that the functional $I = \int_{x_1}^{x_2} (y^2 + x^2 y^1)dx$ assumes extreme values on the straight line y = x.

   c) Define geodesic on a surface.

   d) Evaluate $\int_C [(3x+y)dx + (2y-x)dy]$ along $y = x$ from (0,0) to (10,10).

   e) Evaluate $\int_0^{\frac{\pi}{2}} \int_0^a r^2 dr d\theta$.

   f) Evaluate $\int_0^1 \int_0^2 \int_0^3 (x+y+z)dxdydz$.

   g) State Gauss - Divergence theorem.

   h) Evaluate $\oint_C (yzdx + zxdy + xydz)$ where 'C' is the curve $x^2 + y^2 = 1$, $z = y^2$ using Stoke's theorem.

PART - B

Answer **two** full questions. **(2×10=20)**

2. a) Derive Euler's equation in the form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y^1}\right) = 0$.

   b) Prove that geodesic on a plane is a straight line.

(OR)

3. a) Solve the variational problem $\delta\int_0^{\frac{\pi}{2}}\left[y^2-(y^1)^2\right]dx=0$ under the conditions $y(0)=0,\ y\left(\frac{\pi}{2}\right)=2$.

b) Show that the extremal of $\int_{x_1}^{x_2}\left(\frac{y^1}{y}\right)^2 dx$ can be expressed in the form $y=ae^{bx}$.

4. a) Prove that catenary is the curve which when rotated about a line generates a surface of minimum area.

b) Find the extremal of the functional $\int_{x_1}^{x_2}\left[12xy+(y^1)^2\right]dx$.

(OR)

5. a) Find the extremal of the functional $I=\int_0^1\left[(y^1)^2+x^2\right]dx$ subject to the constraint $\int_0^1 ydx=2$ and having end conditions $y(0)=0, y(1)=1$.

b) Find the function y which makes the integral $I=\int_{x_1}^{x_2}[y^2+4(y^1)^2]dx$ an extremum.

PART - C

Answer **two** full questions. **(2×10=20)**

6. a) Evaluate $\int_C[(x+2y)dx+(4-2x)dy]$ along the curve $C:\frac{x^2}{16}+\frac{y^2}{9}=1$ in anticlockwise direction.

b) Evaluate $\iint_A(4x^2-y^2)dxdy$, where A is the area bounded by the lines $y=0$, $y=x$ and $x=1$.

(OR)

7. a) Evaluate $\int_0^a\int_0^{2\sqrt{ax}} x^2dxdy$ by changing the order of integration.

b) Compute $\int_C(xdx+ydy)$ around the square (0,0), (1,0), (1,1), (0,1).

8. a) Evaluate $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} xyz\,dz\,dy\,dx$.

b) Evaluate $\iint_R \frac{x^2y^2}{x^2+y^2}\,dx\,dy$ by changing into polar coordinates, where R is the annular region between the circles $x^2+y^2=4$ and $x^2+y^2=1$.

(OR)

9. a) Evaluate $\iiint_V xy\,dx\,dy\,dz$ over the region bounded by the coordinate planes and the plane $x+y+z=1$.

b) Find volume of the sphere $x^2+y^2+z^2=a^2$ by triple integration.

PART - D

Answer **two** full questions. **(2×10=20)**

10. a) State and prove Green's theorem.

b) Evaluate using Gauss - Divergence thorem $\iint_S(\vec{F}.\hat{n})\,dS$, where $\vec{F}=(2xy)\hat{i}+(yz^2)\hat{j}+(xz)\hat{k}$ and S is the total surface of the rectangular parallelopiped bounded by the planes x = 0, y = 0, z = 0, x =1, y = 2, z = 3.

(OR)

11. a) Evaluate by Stoke's theorem

$\int_C(\sin z dx-\cos x dy+\sin y dz)$, where 'C' is the boundary of the rectangle $0\le x\le\pi,\ 0\le y\le 1,\ z=3$.

b) Using Green's theorem, evaluate $\oint_C(y^2dx+x^2dy)$, where C is the closed curve bounded by y = x and y^2 = x.

12. a) Using Gauss - Divergence theorem, evaluate $\iint_S\left(x\hat{i}+y\hat{j}+z\hat{k}\right).\hat{n}\,dS$, where S is closed surface bounded by the cone $x^2+y^2=z^2$ and the plane z = 1.

b) Using stoke's theorem, evaluate $\oint_C\vec{F}.d\vec{r}$ where $\vec{F}=(2x-y)\hat{i}-(yz^2)\hat{j}-(y^2z)\hat{k}$ and S is the upper half of the sphere $x^2+y^2+z^2=1$ and C is its boundary.

(OR)

13. a) Using Gauss - Divergence theorem, evaluate $\iint_S (\vec{F}.\hat{n})dS$, where $\vec{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$ over the rectangular parallelepiped $0 \le x \le a, 0 \le y \le b, 0 \le z \le c$.

b) Using Green's theorem, evaluate $\int_C \left[(xy + y^2)dx + x^2 dy\right]$, where 'C' is the closed curve bounded by y = x and $y = x^2$.