11323

Reg. No. | | | | | | | | |

III Semester B.Sc. Degree Examination, April - 2022

MATHEMATICS

(CBCS Semester Scheme 2018 Batch and onwards)

Paper : III

Time : 3 Hours **Maximum Marks : 70**

Instructions to Candidates:

Answer all questions.

PART - A

Answer any **five** questions. **(5×2=10)**

1. a) Define right coset of a subgroup of a group.

   b) Find all the left cosets of the subgroup $H=\{0,2,4\}$ of the group $(Z_6,\oplus_6)$.

   c) State Raabe's test.

   d) Show that $\left\{\frac{1}{n}\right\}$ is monotonically decreasing sequence.

   e) Test the convergence of the series

   $$\sqrt{\frac{1}{4}}+\sqrt{\frac{2}{6}}+\sqrt{\frac{3}{8}}+.....$$

   f) State Cauchy's mean value theorem.

   g) Verify Rolle's theorem for the function $f(x)=8x-x^2$ in [2,6].

   h) Evaluate $\underset{x\to 0}{lt}\left(\frac{1-\cos x}{x^2}\right)$ by using L - Hospitals' rule.

[P.T.O.

PART - B

Answer **one** full question. **(1×15=15)**

2. a) In a group G, prove that $O(a) = O(a^{-1}), \forall a \in G$.

   b) Find all the generators of the cyclic group of order 8.

   c) Find all the distinct right cosets of the subgroup $H = \{0,4,8\}$, in $(Z_{12}, \oplus_{12})$.

(OR)

3. a) Prove that any two right (left) cosets of subgroup H of a group G are either identical or disjoint.

   b) Prove that every group of order Four is abelian.

   c) If an element 'a' of a group G is of order 'n' and e is the identity in G, then prove that for some positive integer m, $a^m = e$ if and only if n divids m.

PART - C

Answer **two** full questions. **(2×15=30)**

4. a) If $\lim_{n\to\infty} a_n = a$ and $\lim_{n\to\infty} b_n = b$, prove that $\lim_{n\to\infty}(a_n + b_n) = a + b$.

   b) Discuss the nature of the sequence $\left\{\left(1+\frac{1}{n}\right)^n\right\}$.

   c) Test the convergence of the sequence

      i. $\left\{\frac{3n-4}{4n+3}\right\}$ ii. $\left\{\frac{(-1)^{n-1}}{n}\right\}$.

(OR)

5. a) Prove that every convergent sequence is bounded.

   b) Examine the convergence of the sequence

      i. $\left\{\left(\frac{2n^2+3n+5}{n+3}\right) Sin\left(\frac{\pi}{n}\right)\right\}$ ii. $\left\{\left(1+\frac{a}{n}\right)^{\frac{n}{b}}\right\}$

   c) Prove that a sequence which is monotonically increasing and bounded above is convergent.

6. a) State and prove D'Alembert's ratio test for series of positive terms.

b) Test the convergence of the series.

$$\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+.....$$

c) Find the sum to infinity of the series $1+2\left(\frac{1}{9}\right)+\frac{2.5}{1.2}\left(\frac{1}{81}\right)+\frac{2.5.8}{1.2.3}\left(\frac{1}{729}\right)+.....$

(OR)

7. a) State and prove Cauchy's root test for a series of positive terms.

b) Discuss convergence of the series

$$\sum\frac{1.2.3.....n}{3.5.7.9.....(2n+1)}.$$

c) Sum to infinity the series

$$1+\frac{1+2}{\lfloor 2}+\frac{1+2+3}{\lfloor 3}+\frac{1+2+3+4}{\lfloor 4}+.....$$

PART - D

Answer **one** full questions. **(1×15=15)**

8. a) Prove that every continuous function over a closed interval is bounded.

b) Evaluate $\underset{x\to 0}{lt}(1+\sin x)^{\cot x}$ using L - Hospital's rule.

c) Expand the function $\log_e^{(1+x)}$ up to the term containing x^4 by Maclaurin's expanison.

(OR)

9. a) State and prove Lagranges mean value theorem.

b) Examine the differentiability of the function $f(x)=\begin{cases}x^2 & if\ x\leq 3\\ 6x-9 & if\ x>3\end{cases}$ at x = 3.

c) Evaluate $\lim_{x\to 0}\left(\frac{1}{x}\right)^{\tan x}$ by using L - Hospital's rule.