Nagarjuna Degree College
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I Semester B.Sc. Degree Examination, April - 2022

MATHEMATICS - I

Paper : I

(CBCS Semester Scheme 2019 Prior to 2019) (Repeater)

Time : 3 Hours **Maximum Marks : 70**

Instructions to Candidates:

Answer all questions.

PART - A

Answer any **five** questions. **(5×2=10)**

1. a) State Cayley - Hamilton theorem.

b) If $A=\begin{bmatrix}2 & -1\\ 0 & 1\end{bmatrix}$, find the characteristic roots of A.

c) Find the n[th] derivative of $Cos(4x+3)$.

d) If $Z=e^{x/y}$ then find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

e) Evaluate $\int_0^{\pi}\sin^4 x\,dx$.

f) Evaluate $\int_{-\pi/2}^{\pi/2}\cos^8 x\,dx$.

g) Find the centre and radius of the sphere $x^2+y^2+z^2-6x+8y-10z+1=0$.

h) Find the angle between the planes $2x+2y-3z-5=0$ and $3x-3y+5z-6=0$.

[P.T.O.

PART - B

Answer one full question. (1×15=15)

2. a) Find the rank of the matrix $\begin{bmatrix} 5 & 3 & 14 & 4 \\ 0 & 1 & 2 & 1 \\ 1 & -1 & 2 & 0 \end{bmatrix}$ by reducing to echelon form.

b) Solve the system of equations by Gauss elimination method $x+y+z=9$, $x-2y+3z=8$ and $2x+y-z=3$.

c) Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 1 & 2 \\ 5 & 4 \end{bmatrix}$.

(OR)

3. a) Reduce the matrix $A=\begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 & -3 & 6 \\ 3 & -3 & 1 & 2 \end{bmatrix}$ to the normal form and hence find its rank.

b) Test for Consistency and solve $5x+y+3z=20$, $2x+5y+2z=18$ and $3x+2y+z=14$.

c) Verify Cayley - Hamilton theorem for the matrix $A=\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$.

PART - C

Answer **two** full questions. **(2×15=30)**

4. a) Find the n[th] derivative of $x^3e^x\sin 2x$ by using Leibnitz's theorem.

b) Find the n[th] derivative of $\frac{1}{6x^2-5x+1}$.

c) If $y=\sin^{-1}x$, show that $\left(1-x^2\right)y_{n+2}-(2n+1)xy_{n+1}-n^2y_n=0$.

(OR)

5. a) If $Z=\sin(ax+y)+\cos(ax-y)$ then prove that $\frac{\partial^2 z}{\partial x^2}=a^2\frac{\partial^2 z}{\partial y^2}$.

b) State and prove the Euler's theorem.

c) If $u=f(x-y,y-z,z-x)$, show that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$.

6. a) If $x = R\cos\phi$, $y = R\sin\phi$ and $Z = Z$ find $\frac{\partial(x,y,z)}{\partial(R,\phi,z)}$.

b) If $u = 2xy$, $v = x^2 - y^2$ and $x = r\cos\theta$, $y = r\sin\theta$ then find $\frac{\partial(u,v)}{\partial(r,\theta)}$.

c) Obtain the reduction formula for $\int \cos^n x dx$, where n is a postive integer.

(OR)

7. a) Evaluate $\int_0^1 x^2(1-x^2)^{3/2} dx$.

b) Show that $\int_0^{\pi} \frac{\sin^4\theta}{(1-\cos\theta)^2} d\theta = \frac{16}{3}$.

c) Evaluate $\int_0^1 \frac{(x^a - 1)}{\log x} dx$, using Leibnitz's rule of differentiation under integral sign.

PART - D

Answer **one** full questions. **(1×15=15)**

8. a) Find the equation of the plane through the intersection of the planes $x - 2y + z - 7 = 0$ and $2x + 3y - 4z = 0$ and cutting intercept 4 units on the x-axis.

b) Find the equation of the sphere which passes through the four points (1,2,3), (0,-2,4), (4,-4,2) and (3,1,4).

c) Find the equation of the right circular cone through the point (2,1,3) with vertex at the point and axis parallel to the line $\frac{x-2}{2} = \frac{y-1}{-4} = \frac{z+2}{3}$.

(OR)

9. a) Find the length and equations of the line of shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-6}{4} = \frac{z-5}{5}$.

b) Derive the equation of the right circular cone in its standard form $x^2 + y^2 = z^2\tan^2\alpha$.

c) Find the equation of the right circular cone whose centre is (2,-3,5), axis makes equal angles with the co-ordinate axes and with semi verticle angle $Cos^{-1}\left(\frac{\sqrt{2}}{3}\right)$.