12125

Reg. No.

I Semester B.Sc. Degree Examination, April - 2022

MATHEMATICS

(CBCS Semester Scheme 2019 Batch and Onwards)

Paper : I

Time : 3 Hours **Maximum Marks : 70**

Instructions to Candidates:

Answer **all** the questions.

I. Answer any **Five** questions. **(5×2=10)**

1. Reduce the matrix $A=\begin{bmatrix}1 & 3 & -2\\ 2 & -1 & 4\\ 1 & -11 & 14\end{bmatrix}$ to echelon form.

2. Define consistency and inconsistency of the system of linear equations.

3. Find the nth derivative of $\log(2x+3)$.

4. If $u=ax^2+2hxy+by^2$ show that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=2u$.

5. Evaluate $\int_0^{\pi/2}\sin^6 x\,dx$

6. Evaluate $\int_0^{\pi/2}\sin^4 x\cos^2 x\,dx$.

7. Find centre and radius of the sphere $x^2+y^2+z^2-6x+8y-10z+1=0$.

8. Find λ if the spheres $x^2+y^2+z^2+6z-\lambda=0$ and $x^2+y^2+z^2+10y-4z-8=0$ cut orthogonally.

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II. Answer any **Three** of the following questions. (3×5=15)

9. Find the rank of the matrix $\begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 2 & -5 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{bmatrix}$ by reducing it to echelon for m.

10. Find rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}$ by reducing it to normal form.

11. Show that the equations $x+y+z=6$, $x+2y+3z=14$, $x+4y+7z=30$ are consistent and solve them.

12. Find the eigen values and the corresponding eigen vectors of the matrix $A=\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.

13. State the prove Cayley - Hamilton theorem.

III. Answer any **three** questions. (3×5=15)

14. Find the n[th] derivative of $e^{ax}\, Cos(bx+c)$.

15. State and prove Leibnitz theorem for finding the n[th] derivative of product of two functions.

16. If $u=(x+y)^n+(y-z)^n+(z-x)^n$ prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$.

17. If $u=\cos^{-1}\left(\frac{x^3+y^3}{x+y}\right)$ prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=-2\cot u$.

18. If $x=r\cos\theta$, $y=r\sin\theta$ prove that $JJ^1=1$ where $J=\frac{\partial(x,y)}{\partial(r,\theta)}$, $J^1=\frac{\partial(r,\theta)}{\partial(x,y)}$.

IV. Answer any **two** questions. (2×5=10)

19. Obtain reduction formula for $\int \tan^n x\,dx$ and evaluate $\int_0^{\pi/4} \tan^n x\,dx$.

20. Evaluate $\int_0^{\pi} x\sin^7 x\,dx$.

21. Evaluate $\int_0^{\pi} \frac{\log(1+\alpha\cos x)}{\cos x}dx$ where α is the parameter using Leibnitz's rule for differentiation under the integral sign.

V. Answer any **two** questions. (2×5=10)

22. Obtain equation of sphere passing through the points (0,0,0), (a,0,0), (0,b,0) and (0,0,c).

23. Find equation of right circular cone whose vertex is the origin and axis is the line $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and having semi - vertical angle of 30°.

24. Find equation of right circular cylinder of radius 3 and axis $\frac{x-1}{2}=\frac{y-3}{2}=\frac{z-5}{-1}$.

VI. Answer any **two** questions. (2×5=10)

25. The sum of three numbers is 26. The third number is twice the second and is also 1 less than 3 times the first. What are the three numbers.

26. A particle of mass 3 units moving along the space curve defined by $\vec{r}=(4t^2+t^3)\hat{i}+5t\,\hat{j}+(t^3+2)\hat{k}$ Find

    i. the momentum.

    ii. force acting on it at t = 2.

27. The population grows at the rate of 5% per year. How long does it take for the population to double.